

Superstrings in type IIB R-R plane-wave in semi-light-cone gauge and conformal invariance

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Superstrings in type IIB R-R plane-wave in semi-light-cone gauge and conformal invariance

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ABSTRACT: We reconsider the analysis done by Kazama and Yokoi in arXiv:0801.1561 (hep-th). We find that although the right vacuum of the theory is the one associated to massless normal ordering (MNO), phase space normal ordering (PNO) plays crucial role in the analysis in the following way. While defining the quantum energy-momentum (EM) tensor one needs to take into account the field redefinition relating the space-time field and the corresponding world-sheet coupling. We argue that for a simple off-shell ansatz for the background this field redefinition can be taken to be identity if the interaction term is ordered according to PNO. This definition reproduces the correct physical spectrum when the background is on-shell. We further show that the right way to extract the effective equation of motion from the Virasoro anomaly is to first order the anomaly terms according to PNO at a finite regularization parameter ϵ and then take the $\epsilon \rightarrow 0$ limit. This prescription fixes an ambiguity in taking the limit for certain bosonic and fermionic contributions to the Virasoro anomaly and is the natural one to consider given the above definition of the EM tensor.

KEYWORDS: Penrose limit and pp-wave background, Conformal Field Models in String Theory, Anomalies in Field and String Theories, Sigma Models

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1 Introduction

Green-Schwarz superstring is usually quantized in light-cone gauge [1], in which case the world-sheet theory does not have conformal invariance. This invariance can however be preserved in semi-light-cone gauge [2–4], where the κ -symmetry is fixed along with the conformal gauge for the bosonic part. A BRST method for this gauge was discussed by Berkovits and Marchioro in [4] for flat background. More recently the same approach was considered in [5] by Kazama and Yokoi (KY) to study type IIB string theory in R-R plane-wave background [6–8]. The main motivation was to establish conformal invariance of the theory.¹ Here we reconsider their analysis to understand certain puzzling issues that will be explained as we go on.

Unlike in light-cone gauge [7, 8], in this case the world-sheet theory is interacting which makes the standard canonical quantization difficult [5] (see also [11]). However, a phase-space (operator) method was used by KY to compute the Virasoro algebra. Since

¹See [9] for other CFT approaches for this background. More general R-R pp-waves have been discussed in [10].

such a computation involves calculating only equal-time commutators, it can in principle be done without solving for time-evolution once all the operators are expressed in terms of the phase-space modes.² However, one needs to choose a normal ordering prescription. The prescription that gives the right theory for flat background has been called massless normal ordering (MNO). In [5] the following result was shown for R-R plane-wave:

If the theory is defined with MNO, then the Virasoro algebra is not satisfied due to non-zero anomalous terms. However, this anomaly vanishes if the theory is defined with a different prescription called phase-space normal ordering (PNO).

(1.1)

It was therefore concluded that PNO is the right choice for R-R plane-wave.

In a previous work [12] we pointed out that such a result is very surprising from the point of view of the universality property of pp-waves as discussed in [13]. Because of this property one expects that the vacuum of the world-sheet theory in a pp-wave background should correspond to the same normal ordering prescription as in flat background. To understand this in a simpler setting we considered the simplest class of off-shell pp-waves in bosonic string theory where only the lower ++ component of the metric is switched on. By defining the theory with MNO it was shown that the Virasoro algebra is satisfied and the correct physical spectrum is reproduced when the background is on-shell.

In this work we address the similar issues in the original context of R-R plane-wave. To this end we consider the same off-shell metric as above along with a constant five-form R-R flux. Because of the flux the effective equation of motion for the background becomes slightly more complicated. This makes the results of [12] partially invalid for the present case in the following sense. Although it is still true that the right vacuum of the theory is the one associated to MNO as expected, definition of the quantum energy-momentum (EM) tensor needs to be modified. Such a modification does not affect the analysis of [12], but it does in the present case where a flux is turned on. Moreover, extracting the effective equation of motion³ from the Virasoro anomaly becomes more subtle. Below we will discuss these two issues in further detail.

We discuss the definition of the quantum EM tensor in section 2. According to this definition the *free part* is ordered according to MNO, but the *interaction part* according to PNO. The reason for this definition can be understood as follows. The fields appearing in the space-time effective theory and the corresponding world-sheet couplings are in general related by field redefinition. We argue that for the restricted off-shell ansatz considered here this field redefinition can be taken to be identity if the interaction term is ordered according to PNO. One is free to order this term according to MNO as well, but in that case a non-trivial field redefinition needs to be considered. We show in subsection 2.2 that with this definition the correct physical spectrum is reproduced for the on-shell R-R plane-wave.

²The equal-time commutators/anti-commutators of the full set of phase-space modes leads to an algebraic structure that can be used to define a Hilbert space and is same in both the flat background and the R-R plane-wave.

³See [14–17] for a partial list of original references on this subject.

We discuss the computation of Virasoro anomaly in section 3. There are three anomaly terms denoted by $\mathcal{A}^R(\sigma, \sigma')$, $\mathcal{A}^L(\sigma, \sigma')$ and $\mathcal{A}(\sigma, \sigma')$ resulting from the equal-time commutators $[\mathcal{T}(\sigma), \mathcal{T}(\sigma')]$, $[\tilde{\mathcal{T}}(\sigma), \tilde{\mathcal{T}}(\sigma')]$ and $[\mathcal{T}(\sigma), \tilde{\mathcal{T}}(\sigma')]$ respectively. Here σ is the world-sheet space coordinate⁴ and $\mathcal{T}(\sigma)$ and $\tilde{\mathcal{T}}(\sigma)$ are the right and left moving EM tensor components respectively. In the present method of computation each of the above anomaly terms receives contribution from an infinite number of phase-space modes and the actual computation is done in a regularized theory with a finite cutoff ϵ . We show that certain bosonic and fermionic contributions to the anomaly terms develop an ambiguity when we take the $\epsilon \rightarrow 0$ limit. This ambiguity is related to whether we order the operators according to MNO or PNO at a finite cutoff. It turns out that the correct equation of motion emerges in a satisfactory manner if we order the terms according to PNO before taking the limit. This is also required by the definition of EM tensor discussed above.

It is important to note that consideration of PNO as discussed above is not related to the choice of vacuum. Although the interaction term in the EM tensor is ordered according to PNO, the correct spectrum is reproduced when we define the vacuum to be the one associated to MNO. In fact the spectrum contains negative dimensions, as expected [12], if we choose the PNO-vacuum instead (see appendix E). In the context of Virasoro algebra let us consider the fermionic contribution to the Virasoro anomaly. It can be shown using an alternative argument, which does not require us to order the terms in any particular way, that the result is same as obtained in PNO-prescription even in the MNO-vacuum.

We conclude in section 4 and keep some technical details in various appendices. Some of the technical results described here were already known in [5]. However, our analysis and interpretations are very different.

2 Energy-momentum tensor

2.1 Definition and field redefinition

The relevant details of the semi-light-cone quantization and the regularization procedure have been described in appendix A and B respectively from where we will borrow various notations for the discussion to follow. We define the EM tensor in the usual manner: $T_{ab} = -\frac{4\pi}{\sqrt{-g}} \frac{\delta S}{\delta g^{ab}}$. The right and left moving components are given by: $\mathcal{T} = \frac{1}{2}(T_{00} - T_{01})$ and $\tilde{\mathcal{T}} = \frac{1}{2}(T_{00} + T_{01})$ respectively. The quantum operators are given by the following expressions:

$$\mathcal{T} = \mathcal{T}^{(0)} + \delta\mathcal{T}, \quad \tilde{\mathcal{T}} = \tilde{\mathcal{T}}^{(0)} + \delta\tilde{\mathcal{T}}, \quad (2.1)$$

where the *free parts* relevant to the flat background are given by [4] (see also [3]),⁵

$$\begin{aligned} \mathcal{T}^{(0)} &= : \left(\frac{1}{2} \Pi \cdot \Pi - \frac{i}{2} (S \partial_\sigma S) + \xi \partial_\sigma^2 \ln \Pi^+ \right) : +1, \\ \tilde{\mathcal{T}}^{(0)} &= : \left(\frac{1}{2} \tilde{\Pi} \cdot \tilde{\Pi} + \frac{i}{2} (\tilde{S} \partial_\sigma \tilde{S}) + \xi \partial_\sigma^2 \ln \tilde{\Pi}^+ \right) : +1, \end{aligned} \quad (2.2)$$

⁴Throughout the paper we will suppress the world-sheet time coordinate τ in the arguments. All our analysis are understood to be done at the same τ .

⁵We will use the notations: $A \cdot B = \eta_{\mu\nu} A^\mu B^\nu$ and $\vec{A} \cdot \vec{B} = A^I B^I$.

where : : refers to MNO. The last terms inside the round brackets arise from non-covariant gauge fixing. The normal ordering constant 1 and $\xi = -\frac{1}{2}$ have been fixed in appendix C from the condition that $\mathcal{T}^{(0)}$ and $\tilde{\mathcal{T}}^{(0)}$ satisfy the standard form of the Virasoro algebra with central charge 26. This cancels with -26 coming from the (b, c) ghost system which turns out to be the only propagating ghost degrees of freedom. The *interaction part* is given by: $\delta\mathcal{T} = \delta\mathcal{T}_B + \delta\mathcal{T}_F$ where,

$$\delta\mathcal{T}_B = -\frac{1}{2}\chi^2 K(\vec{X}), \quad \delta\mathcal{T}_F = \frac{i\mu}{2\sqrt{\pi T}} \chi(S\Sigma\tilde{S}), \quad (2.3)$$

where $\chi(\sigma) = \sqrt{\Pi^+\tilde{\Pi}^+(\sigma)}$, $\Sigma = \sigma^{1234}$ and,

$$K(\vec{x}) = -\mu^2 \vec{x}^2. \quad (2.4)$$

As mentioned earlier the interaction part is ordered according to PNO. For both the bosonic and fermionic interactions in (2.3) PNO gives rise to operators without any non-trivial re-ordering of the modes. Given the anti-commutators below eqs. (A.5), this is obvious for $\delta\mathcal{T}_F$. For $\delta\mathcal{T}_B$ see the discussion below eqs. (B.4).

The above definition can be understood to relate, in a particular way, the field in the space-time effective theory and the corresponding world-sheet coupling. To see that more explicitly let us consider an off-shell ansatz for the background where the R-R flux is still constant, but the metric is given by,

$$ds^2 = 2dx^+dx^- + K(\vec{x})(dx^+)^2 + d\vec{x}.d\vec{x}, \quad (2.5)$$

where $K(\vec{x})$ is arbitrary. This background is characterized by the Fourier transform $\tilde{K}(\vec{p}) = \int \frac{d\vec{x}}{(2\pi)^{d/2}} K(\vec{x})e^{-i\vec{p}.\vec{x}}$. According to our definition the same Fourier transform is used as the world-sheet coupling in $\delta\mathcal{T}_B$ in (2.3), i.e.

$$K(\vec{X}(\sigma)) = \int \frac{d\vec{p}}{(2\pi)^{d/2}} \tilde{K}(\vec{p})e^{i\vec{p}.\vec{X}(\sigma)}, \quad (2.6)$$

provided the exponential operator is unordered (i.e. ordered according to PNO). We could alternatively define the above operator according to MNO,

$$K(\vec{X}(\sigma)) = \int \frac{d\vec{p}}{(2\pi)^{d/2}} \tilde{K}'(\vec{p}) : e^{i\vec{p}.\vec{X}(\sigma)} :. \quad (2.7)$$

But in that case the Fourier transform $\tilde{K}'(\vec{p})$ has to be identified with the one characterizing the background (2.5) up to the following field redefinition,

$$\tilde{K}'(\vec{p}) = e^{-\frac{p^2}{2}D_\epsilon(0)} \tilde{K}(\vec{p}), \quad (2.8)$$

where ϵ is the regularization parameter and $\eta^{\mu\nu}D_\epsilon(\Delta)$ is the bosonic “equal time propagator” as discussed in appendix B. This relation is simply obtained by equating the operators in (2.6) and (2.7) and using $e^{i\vec{p}.\vec{X}(\sigma)} = e^{-\frac{p^2}{2}D_\epsilon(0)} : e^{i\vec{p}.\vec{X}(\sigma)} :$.

2.2 Physical spectrum

Given the above definition of the quantum EM tensor we will now compute the physical spectrum for the quadratic profile in (2.4) and show that it reproduces the right answer. We will follow the method of [12] which requires us to have a CFT formulation. Here we will simply assume the conformal invariance which will be established in the next section.

Unless stated otherwise we will mostly follow the notations of [12]. We take the quadratic space-time action to be as given in eq. (2.25)⁶ where the inner product is understood to be the hermitian inner product. The orthonormal basis states spanning the transverse Hilbert space \mathcal{H}_\perp are given by $|\{N\}, p, \eta\rangle = c_1 \tilde{c}_1 |\{N\}, p, \eta\rangle$ such that $|\{N\}, p, \eta\rangle$ satisfies the conditions [2.21]. Here $\{N\}$ represents a set of four sets of integers, namely: $\{\{N_{\text{In}}^B\}, \{\tilde{N}_{\text{In}}^B\}, \{N_{\text{an}}^F\}, \{\tilde{N}_{\text{an}}^F\}\}$ such that $|\{N\}, p, \eta\rangle$ is proportional to:

$$\prod_{n>0, I, a} (\Pi_{-n}^I)^{N_{\text{In}}^B} (\tilde{\Pi}_{-n}^I)^{\tilde{N}_{\text{In}}^B} (S_{-n}^a)^{N_{\text{an}}^F} (\tilde{S}_{-n}^a)^{\tilde{N}_{\text{an}}^F} |p, \eta\rangle, \quad (2.9)$$

where the momentum p^μ of the ground states $|p, \eta\rangle = e^{ip \cdot X_0} |\eta\rangle$ is normalized as: $\Pi_0^\mu |p, \eta\rangle = \frac{p^\mu}{2\sqrt{\pi T}} |p, \eta\rangle$. From the linearized equation of motion one obtains the following eigenvalue equation:

$$\mathcal{S}_2 |\Psi_\perp\rangle = 0, \quad (2.10)$$

where,

$$\mathcal{S}_2 = L_0 + \tilde{L}_0 - 2, \quad |\Psi_\perp\rangle = \sum'_{n, \{N\}} \int dp \tilde{\psi}_{n, \{N\}}(p) |\{N\}, p, \eta\rangle. \quad (2.11)$$

The prime on the summation refers to the condition $(L_0 - \tilde{L}_0) = 0$. The normal ordering constant in L_0 and \tilde{L}_0 has been fixed in appendix C which precisely cancels the ghost contribution of -2 in the first equation of (2.11). Using this we get the following result inside \mathcal{H}_\perp :

$$\begin{aligned} \mathcal{S}_2 &= \mathcal{S}_2^0 + \mathcal{S}_2^{\neq 0}, \\ \mathcal{S}_2^0 &= \frac{\alpha'}{2} \left[p^2 + (\mu p^+)^2 X_0^I X_0^I + 2i\mu p^+ (S_0 \Sigma \tilde{S}_0) \right], \\ \mathcal{S}_2^{\neq 0} &= \sum_{n>0} \left[\Pi_{-n}^I \Pi_n^I + \tilde{\Pi}_{-n}^I \tilde{\Pi}_n^I + n(S_{-n} S_n) + n(\tilde{S}_{-n} \tilde{S}_n) \right] \\ &\quad + \sum_{n \neq 0} \left[\frac{m^2}{4n^2} (\Pi_{-n}^I \Pi_n^I + \tilde{\Pi}_{-n}^I \tilde{\Pi}_n^I) - \frac{m^2}{2n^2} \Pi_n^I \tilde{\Pi}_n^I + im(S_n \Sigma \tilde{S}_n) \right], \end{aligned} \quad (2.12)$$

where $m = \alpha' \mu p^+$. To diagonalize the bosonic part of $\mathcal{S}_2^{\neq 0}$ we first define the operators α_n^I and $\tilde{\alpha}_n^I$ ($n \neq 0$) precisely in the same way as in [12] by replacing $m_I \rightarrow m$ in eqs.[2.30]. For the fermionic part one defines U_n and \tilde{U}_n ($n \neq 0$) in the following way:

$$S_n = c_n \left(U_n - i \frac{w_n - n}{m} \Sigma \tilde{U}_{-n} \right), \quad \tilde{S}_n = c_n \left(\tilde{U}_n + i \frac{w_n - n}{m} \Sigma U_{-n} \right), \quad (2.13)$$

⁶Equation numbers kept in square brackets will refer to equations from [12].

where $w_n = n\sqrt{1 + \frac{m^2}{n^2}}$ and $c_n = \frac{1}{\sqrt{1 + (\frac{w_n - n}{m})^2}}$. With this definition one can establish: $\{U_m^a, U_n^b\} = \{\tilde{U}_m^a, \tilde{U}_n^b\} = \delta^{ab}\delta_{m+n,0}$, $\{U_m^a, \tilde{U}_n^b\} = 0$. Next we define new vacua $|p, \eta\rangle$ such that $(\alpha_n^I, \tilde{\alpha}_n^I, U_n^a, \tilde{U}_n^a)|p, \eta\rangle = 0$, $\forall n > 0$, and a new basis $|\{N\}, p, \eta\rangle$ which can be obtained by replacing Π and S oscillators in the expression (2.9) by the corresponding α and U oscillators respectively. Expanding $|\Psi_\perp\rangle$ in this new basis one gets the following expected result [8]:

$$\mathcal{S}_2^{\neq 0} = \sum_{n>0} \left[\alpha_{-n}^I \alpha_n^I + \tilde{\alpha}_{-n}^I \tilde{\alpha}_n^I + w_n \left(U_{-n}^a U_n^a + \tilde{U}_{-n}^a \tilde{U}_n^a \right) \right]. \quad (2.14)$$

Such a change of basis produces c-number contributions of $d \sum_{n>0} (w_n - n)$ each for the bosonic and the fermionic parts but with opposite signs so that they cancel each other.

We mainly followed [12] for the above derivation. However, notice that unlike the definition considered here, $\delta\mathcal{T}_B$ was ordered according to MNO in that work. Had we done the same thing here, the bosonic part of the last line in (2.12) would have been replaced by,

$$\begin{aligned} & \sum_{n>0, I} \frac{m^2}{2n^2} \left(\Pi_{-n}^I \Pi_n^I + \tilde{\Pi}_{-n}^I \tilde{\Pi}_n^I - \Pi_n^I \tilde{\Pi}_n^I - \tilde{\Pi}_{-n}^I \Pi_{-n}^I \right), \\ & = \left[\sum_{n \neq 0, I} \frac{m^2}{4n^2} \left(\Pi_{-n}^I \Pi_n^I + \tilde{\Pi}_{-n}^I \tilde{\Pi}_n^I \right) - \frac{m^2}{2n^2} \Pi_n^I \tilde{\Pi}_n^I \right] + Z(m^2), \end{aligned} \quad (2.15)$$

where the term in the square bracket is the operator ordered with PNO as considered here. But there is an additional contribution,

$$Z(m^2) = -\frac{dm^2}{2} \sum_{n>0} \frac{1}{n}, \quad (2.16)$$

which can easily be recondensed as⁷ $d\mu^2 D_\epsilon(0) \oint \chi(\sigma)^2$ evaluated inside \mathcal{H}_\perp . This additional negative contribution (leading to a wrong spectrum) is coming due to the fact that we have wrongly identified $\tilde{K}'(\vec{p})$ in eq. (2.7) (instead of $\tilde{K}(\vec{p})$ in eq. (2.6)) with the Fourier transform $(2\pi)^{d/2} \mu^2 \partial_{\vec{p}} \cdot \partial_{\vec{p}} \delta(\vec{p})$ which corresponds to the quadratic profile (2.4). This is precisely the reason why the (unwanted) factor $Z(m^2) = -\frac{1}{2} (\sum_I m_I^2) \sum_{n>0} \frac{1}{n}$ appeared in the zero-point energy in eq.[2.32]. In that case this factor is zero as $\sum_I m_I^2 = 0$ because of the simple on-shell condition in [2.3] (see also [2.14], [2.15]). However, in a more complicated background, such as the one considered in this paper where a flux is turned on, such a contribution is not zero.

3 Virasoro anomaly

3.1 Equation of motion

In this section we will discuss the computation of Virasoro algebra. In particular, we will ask how to extract, from the Virasoro anomalies, the supergravity equation of motion which, for the present off-shell ansatz, is simply given by,

$$-\frac{1}{2} \vec{\partial}^2 K = d\mu^2. \quad (3.1)$$

⁷We will use the notation \oint for the definite integral $\int_0^{2\pi} \frac{d\sigma}{2\pi}$.

It is only when the above equation is satisfied we would expect all the three equal time commutators $[\mathcal{T}(\sigma), \mathcal{T}(\sigma')]$, $[\tilde{\mathcal{T}}(\sigma), \tilde{\mathcal{T}}(\sigma')]$ and $[\mathcal{T}(\sigma), \tilde{\mathcal{T}}(\sigma')]$ to satisfy the standard expressions for the Virasoro algebra (see for example [12]).⁸ However for a generic off-shell configuration each one is supposed to contain an anomaly term. Following [12] we use the Virasoro algebra satisfied by $\mathcal{T}^{(0)}(\sigma)$ and $\tilde{\mathcal{T}}^{(0)}(\sigma)$ to derive the following expressions for these anomaly terms:

$$\begin{aligned} \mathcal{A}^R(\sigma, \sigma') &= A^R(\sigma, \sigma') + [\delta\mathcal{T}_F(\sigma), \delta\mathcal{T}_F(\sigma')], \\ \mathcal{A}^L(\sigma, \sigma') &= A^L(\sigma, \sigma') + [\delta\mathcal{T}_F(\sigma), \delta\mathcal{T}_F(\sigma')], \\ \mathcal{A}(\sigma, \sigma') &= A(\sigma, \sigma') + [\delta\mathcal{T}_F(\sigma), \delta\mathcal{T}_F(\sigma')], \end{aligned} \tag{3.2}$$

where we have used: $[\delta\mathcal{T}_B(\sigma), \delta\mathcal{T}_B(\sigma')] = 0$ (see [12]) and,

$$\begin{aligned} A^R(\sigma, \sigma') &= [\mathcal{T}^{(0)}(\sigma), \delta\mathcal{T}(\sigma')] + [\delta\mathcal{T}(\sigma), \mathcal{T}^{(0)}(\sigma')] + 4\pi i \delta\mathcal{T}(\sigma) \delta'(\Delta) + 2\pi i \partial_\sigma \delta\mathcal{T}(\sigma) \delta(\Delta), \\ A^L(\sigma, \sigma') &= [\tilde{\mathcal{T}}^{(0)}(\sigma), \delta\mathcal{T}(\sigma')] + [\delta\mathcal{T}(\sigma), \tilde{\mathcal{T}}^{(0)}(\sigma')] - 4\pi i \delta\mathcal{T}(\sigma) \delta'(\Delta) - 2\pi i \partial_\sigma \delta\mathcal{T}(\sigma) \delta(\Delta), \\ A(\sigma, \sigma') &= [\tilde{\mathcal{T}}^{(0)}(\sigma), \delta\mathcal{T}(\sigma')] + [\delta\mathcal{T}(\sigma), \tilde{\mathcal{T}}^{(0)}(\sigma')], \end{aligned} \tag{3.3}$$

where $\Delta = \sigma - \sigma'$. We will now show that the condition that all the anomaly terms in (3.2) be zero is same as (3.1). We borrow the following results derived in subsection 3.2:

$$A^R(\sigma, \sigma') = A^L(\sigma, \sigma') = A(\sigma, \sigma') = \frac{i}{4T} \left(2\bar{\partial}^2 \delta\mathcal{T}_B(\sigma) \delta'(\Delta) + \partial_\sigma \bar{\partial}^2 \delta\mathcal{T}_B(\sigma) \delta(\Delta) \right), \tag{3.4}$$

and

$$[\delta\mathcal{T}_F(\sigma), \delta\mathcal{T}_F(\sigma')] = -\frac{i}{4T} \left(2d\mu^2 \chi^2(\sigma) \delta'(\Delta) + d\mu^2 \partial_\sigma \chi^2(\sigma) \delta(\Delta) \right), \tag{3.5}$$

such that,

$$\mathcal{A}^R(\sigma, \sigma') = \mathcal{A}^L(\sigma, \sigma') = \mathcal{A}(\sigma, \sigma') = \frac{i}{4T} \left[2E(\sigma) \delta'(\Delta) + \partial_\sigma E(\sigma) \delta(\Delta) \right], \tag{3.6}$$

where,

$$E(\sigma) = \bar{\partial}^2 \delta\mathcal{T}_B(\sigma) - d\mu^2 \chi^2(\sigma). \tag{3.7}$$

Using $\bar{\partial}^2 \delta\mathcal{T}_B = -\frac{1}{2} \chi^2 \bar{\partial}^2 K$ we conclude that vanishing of the anomaly terms implies,

$$-\frac{1}{2} \bar{\partial}^2 K(\sigma) = d\mu^2, \quad \text{i.e.} \quad \frac{1}{2} \vec{k}^2 \tilde{K}(\vec{k}) = (2\pi)^{d/2} d\mu^2 \delta(\vec{k}), \tag{3.8}$$

where the last equation is the Fourier transform of (3.1).

⁸The definitions of $\mathcal{T}(\mathcal{T}^{(0)})$ and $\tilde{\mathcal{T}}(\tilde{\mathcal{T}}^{(0)})$ considered here are interchanged with respect to that in [12].

3.2 Ambiguity in computing Virasoro anomaly and PNO - prescription

The operator $E(\sigma)$ in eq. (3.7) receives contribution from two sources - the first term i.e. the bosonic contribution comes from the anomaly terms in eqs(3.4) and the second term comes from the commutator $[\delta\mathcal{T}_F(\sigma), \delta\mathcal{T}_F(\sigma')]$ in the fermionic sector. As mentioned in appendix B, we do the actual calculations in the regularized theory with a finite cutoff ϵ and take the $\epsilon \rightarrow 0$ limit at the end. Here we will show that following this procedure one encounters ambiguities in computing certain contributions both in the bosonic and the fermionic sectors. This ambiguity originates from the fact that ordering operators according to MNO and PNO in the cutoff theory and then taking the $\epsilon \rightarrow 0$ limit produces different answers. It turns out that the latter, which is the natural one to follow given our definition of the EM tensor, gives rise to the right space-time equation of motion. Below we will consider the two sectors separately.

Bosonic sector. We have shown in appendix D that given the expressions in (3.3), the bosonic anomaly terms take the following forms:

$$\begin{aligned} A^R(\sigma, \sigma') &= C^B(\sigma, \sigma') - C^B(\sigma', \sigma), & A^L(\sigma, \sigma') &= \tilde{C}^B(\sigma, \sigma') - \tilde{C}^B(\sigma', \sigma), \\ A(\sigma, \sigma') &= C^B(\sigma, \sigma') - \tilde{C}^B(\sigma', \sigma) + \pi i \chi^2(\sigma) \partial_\sigma K(\vec{X}(\sigma)) \delta(\Delta), \end{aligned} \quad (3.9)$$

where,

$$C^B(\sigma, \sigma') = [t(\sigma), \delta\mathcal{T}_B(\sigma')], \quad \tilde{C}^B(\sigma, \sigma') = [\tilde{t}(\sigma), \delta\mathcal{T}_B(\sigma')], \quad (3.10)$$

where $t(\sigma)$ and $\tilde{t}(\sigma)$ are defined in eqs. (D.1). Therefore the relevant commutators that we need to compute at finite ϵ are given by:

$$\begin{aligned} \left[: \Pi^I \Pi^I(\sigma) :, e^{i\vec{k} \cdot \vec{X}(\sigma')} \right]_\epsilon &= \sqrt{\frac{\pi}{T}} \delta_\epsilon(\Delta) \left(\Pi_\epsilon^I(\sigma) e_\epsilon^{i\vec{k} \cdot \vec{X}(\sigma')} + e_\epsilon^{i\vec{k} \cdot \vec{X}(\sigma')} \Pi_\epsilon^I(\sigma) \right), \\ \left[: \tilde{\Pi}^I \tilde{\Pi}^I(\sigma) :, e^{i\vec{k} \cdot \vec{X}(\sigma')} \right]_\epsilon &= \sqrt{\frac{\pi}{T}} \delta_\epsilon(\Delta) \left(\tilde{\Pi}_\epsilon^I(\sigma) e_\epsilon^{i\vec{k} \cdot \vec{X}(\sigma')} + e_\epsilon^{i\vec{k} \cdot \vec{X}(\sigma')} \tilde{\Pi}_\epsilon^I(\sigma) \right). \end{aligned} \quad (3.11)$$

Notice that $A^R(\sigma, \sigma')$ and $A^L(\sigma, \sigma')$ in (3.9) have anti-symmetrization in σ and σ' . Therefore because of the $\delta_\epsilon(\Delta)$ factor in eqs. (3.11), classically these anomalies are zero in the $\epsilon \rightarrow 0$ limit as expected. However, in the quantum theory we are supposed to first order the right hand sides of (3.11) according to some normal ordering prescription before taking the limit. We will see that the final result is different if we order the operators according to MNO and PNO.

Let us first consider the right moving sector. In our convention the two normal ordering prescriptions act in the same way in this sector and therefore there is no ambiguity. The result can be written as:

$$\begin{aligned} \Pi_\epsilon^I(\sigma) e_\epsilon^{i\vec{k} \cdot \vec{X}(\sigma')} + e_\epsilon^{i\vec{k} \cdot \vec{X}(\sigma')} \Pi_\epsilon^I(\sigma) &= \frac{k^I}{2\sqrt{\pi T}} \left(d(q, \epsilon) - d(q^{-1}, \epsilon) + 1 \right) e_\epsilon^{i\vec{k} \cdot \vec{X}(\sigma')} \\ &\quad + 2 \left[\Pi_{(-)}^I(\sigma - i\epsilon/2) e_\epsilon^{i\vec{k} \cdot \vec{X}(\sigma')} + e_\epsilon^{i\vec{k} \cdot \vec{X}(\sigma')} \Pi_{(+)}^I(\sigma + i\epsilon/2) \right], \end{aligned} \quad (3.12)$$

where the notation $d(q, \epsilon)$ has been explained below eqs. (B.1). The term in the square bracket is the ordered part and will drop out when we anti-symmetrize and take $\epsilon \rightarrow 0$ limit. To find the contribution of the first term one uses the following identity [5]:⁹

$$\delta_\epsilon(\Delta) (d(q, \epsilon) - d(q^{-1}, \epsilon)) = -i\delta'_\epsilon(\Delta) . \quad (3.14)$$

Using these results and the identity (D.4) in (3.9) one finally gets the result for $A^R(\sigma, \sigma')$ in (3.4).

Let us now consider $A^L(\sigma, \sigma')$. MNO and PNO act differently in this sector. Going through the similar procedure as above one gets the following expressions in the two cases,

$$\begin{aligned} A^L(\sigma, \sigma') &\xrightarrow{\text{MNO}} \int \frac{d\vec{k}}{(2\pi)^{d/2}} \tilde{K}(\vec{k}) \left[\frac{\vec{k}^2}{2T} i\delta'_\epsilon(\Delta) \left(e^{i\vec{k} \cdot \vec{X}}(\sigma') \chi^2(\sigma') + e^{i\vec{k} \cdot \vec{X}}(\sigma) \chi^2(\sigma) \right) \right. \\ &\quad + 2\sqrt{\frac{\pi}{T}} \delta_\epsilon(\Delta) k^I \left\{ \tilde{\Pi}_{(-)}^I(\sigma + i\epsilon/2) e^{i\vec{k} \cdot \vec{X}}(\sigma') \chi^2(\sigma') + e^{i\vec{k} \cdot \vec{X}}(\sigma') \chi^2(\sigma') \tilde{\Pi}_{(+)}^I(\sigma - i\epsilon/2) \right. \\ &\quad \left. \left. - \tilde{\Pi}_{(-)}^I(\sigma' + i\epsilon/2) e^{i\vec{k} \cdot \vec{X}}(\sigma) \chi^2(\sigma) - e^{i\vec{k} \cdot \vec{X}}(\sigma) \chi^2(\sigma) \tilde{\Pi}_{(+)}^I(\sigma' - i\epsilon/2) \right\} \right] , \\ &\xrightarrow{\text{PNO}} \int \frac{d\vec{k}}{(2\pi)^{d/2}} \tilde{K}(\vec{k}) \left[-\frac{\vec{k}^2}{2T} i\delta'_\epsilon(\Delta) \left(e^{i\vec{k} \cdot \vec{X}}(\sigma') \chi^2(\sigma') + e^{i\vec{k} \cdot \vec{X}}(\sigma) \chi^2(\sigma) \right) \right. \\ &\quad + 2\sqrt{\frac{\pi}{T}} \delta_\epsilon(\Delta) k^I \left\{ \tilde{\Pi}_{(+)}^I(\sigma - i\epsilon/2) e^{i\vec{k} \cdot \vec{X}}(\sigma') \chi^2(\sigma') + e^{i\vec{k} \cdot \vec{X}}(\sigma') \chi^2(\sigma') \tilde{\Pi}_{(-)}^I(\sigma + i\epsilon/2) \right. \\ &\quad \left. \left. - \tilde{\Pi}_{(+)}^I(\sigma' - i\epsilon/2) e^{i\vec{k} \cdot \vec{X}}(\sigma) \chi^2(\sigma) - e^{i\vec{k} \cdot \vec{X}}(\sigma) \chi^2(\sigma) \tilde{\Pi}_{(-)}^I(\sigma' + i\epsilon/2) \right\} \right] , \end{aligned} \quad (3.15)$$

where we have already taken $\epsilon \rightarrow 0$ limit for some terms where it can be done unambiguously. Notice that the two expressions on the right hand sides of (3.15) are equal at a finite ϵ . This can be proved as an identity. The only difference is that we have ordered infinite number of terms in two different ways. This is why the overall coefficients for the terms kept in the round brackets differ by a sign in the two cases. However, in the $\epsilon \rightarrow 0$ limit σ and σ' coincide and the four terms kept in the curly brackets cancel each other in both the cases leading to two different results which differ by a sign. Therefore depending on which expression we apply the $\epsilon \rightarrow 0$ limit to we get the following results for the anomaly:

$$\begin{aligned} A^L(\sigma, \sigma') &\xrightarrow{\text{MNO}} -\frac{i}{4T} \left(2\vec{\partial}^2 \delta \mathcal{T}_B(\sigma) \delta'(\Delta) + \partial_\sigma \vec{\partial}^2 \delta \mathcal{T}_B(\sigma) \delta(\Delta) \right) , \\ &\xrightarrow{\text{PNO}} \frac{i}{4T} \left(2\vec{\partial}^2 \delta \mathcal{T}_B(\sigma) \delta'(\Delta) + \partial_\sigma \vec{\partial}^2 \delta \mathcal{T}_B(\sigma) \delta(\Delta) \right) . \end{aligned} \quad (3.16)$$

Comparing the result for $A^R(\sigma, \sigma')$ (in (3.4)) and (3.16) we see that for PNO $A^R(\sigma, \sigma')$ and $A^L(\sigma, \sigma')$ are same, but for MNO they differ by a sign. Therefore according to the first two equations in (3.2) conformal invariance would require us to have $[\delta \mathcal{T}_F(\sigma), \delta \mathcal{T}_F(\sigma')] = 0$ if we adopt the MNO-prescription leading to a wrong space-time equation of motion. We will see later in this section that similar ambiguity exists in the fermionic sector and the MNO-prescription there will indeed dictate this result.

⁹One way to derive this identity is to use (B.4) and the result,

$$\delta_\epsilon(\Delta) \partial_\Delta D_\epsilon(\Delta) = \left(\frac{1}{\pi} \frac{\epsilon}{\Delta^2 + \epsilon^2} \right) \left(-\frac{1}{2\pi T} \frac{\Delta}{\Delta^2 + \epsilon^2} \right) = \frac{1}{4\pi T} \delta'_\epsilon(\Delta) . \quad (3.13)$$

Considering the third anomaly term in (3.4) tells us that PNO-prescription is in fact the natural one to consider given our definition of the EM tensor. In this case the analogue of the terms in the curly brackets in (3.15) do not cancel each other, rather it gives an ordered product of $k^I(\Pi^I - \tilde{\Pi}^I)$ and the exponential operator. Using eqs. (A.4) such operators can be written as $\partial_\sigma e^{i\vec{k}\cdot\vec{X}}(\sigma)$. It is only when we consider PNO such a term cancels with the second term of the last equation in (3.9) which is already defined with PNO.¹⁰ Because of this cancelation the final result turns out to be the one given in eqs. (3.4).

Fermionic sector. Let us now turn to the fermionic sector. We will show that one encounters a similar ambiguity in computing the commutator in (3.5). To see how this ambiguity arises let us first define,

$$F(\sigma) = S(\sigma)\Sigma\tilde{S}(\sigma), \tag{3.17}$$

such that, $\delta\mathcal{T}_F = \frac{i\mu}{2\sqrt{\pi T}}\chi F$. Since χ behaves as a c-number for this computation, we essentially need to calculate the commutator of $F(\sigma)$. The way it was calculated in [5] (which is analogous to the method discussed in the bosonic sector above) is to first write:

$$[F(\sigma), F(\sigma')] = \Sigma_{ab}\Sigma_{cd} \left[\{\tilde{S}^b(\sigma), \tilde{S}^d(\sigma')\} S^c(\sigma') S^a(\sigma) - \{S^a(\sigma), S^c(\sigma')\} \tilde{S}^b(\sigma) \tilde{S}^d(\sigma') \right], \tag{3.18}$$

then to use the anti-commutator in (A.2) to deduce,

$$[F(\sigma), F(\sigma')] = \delta(\sigma - \sigma') \left[S^a(\sigma') S^a(\sigma) - \tilde{S}^a(\sigma) \tilde{S}^a(\sigma') \right]. \tag{3.19}$$

Notice that, just like in the bosonic case, the presence of the delta function forces the fermions to be coincident. Therefore each of the terms in the square bracket drops out classically. But quantum mechanically it is only a normal ordered product that is supposed to vanish leaving a potential c-number contribution. One gets the following results for the two prescriptions at a finite ϵ ,

$$[F(\sigma), F(\sigma')]_\epsilon \xrightarrow{\text{MNO}} -\delta_\epsilon(\sigma - \sigma') \left[: S_\epsilon^a(\sigma) S_\epsilon^a(\sigma') : + : \tilde{S}_\epsilon^a(\sigma) \tilde{S}_\epsilon^a(\sigma') : \right], \\ \xrightarrow{\text{PNO}} -\delta_\epsilon(\sigma - \sigma') \left[\times_\times S_\epsilon^a(\sigma) S_\epsilon^a(\sigma') \times_\times + \times_\times \tilde{S}_\epsilon^a(\sigma) \tilde{S}_\epsilon^a(\sigma') \times_\times \right] + id\delta'_\epsilon(\sigma - \sigma'). \tag{3.20}$$

These results can easily be checked by using eqs. (B.6) and (3.13).

Just like in the bosonic case the equality of the two right hand sides in (3.20) is an algebraic identity. However, in the $\epsilon \rightarrow 0$ limit each of the terms kept in the square brackets goes to zero as they are normal ordered coincident fermions. This leads to two different results:

$$[F(\sigma), F(\sigma')] \xrightarrow{\text{MNO}} 0, \\ \xrightarrow{\text{PNO}} id\delta'(\Delta). \tag{3.21}$$

Using the PNO-prescription and the identity (D.4) one derives the expected result in (3.5).

¹⁰Notice that irrespective of which normal ordering we consider in expressions like (3.15), the exponential operators are always ordered according to PNO. This is because of the particular way the quantum EM tensor has been defined here.

As mentioned earlier, the fact that we get different results following the MNO and PNO-prescriptions in the above method of computation actually reflects an ambiguity and is not an artifact of different choices of vacuum. This can be seen very easily in the fermionic sector by doing the calculation in an alternative method.¹¹ In this method we first define the modes of $F(\sigma)$ as follows:

$$F_n \equiv \oint F(\sigma)e^{-in\sigma} = \sum_{m \in Z} \left(S_m \Sigma \tilde{S}_{m-n} \right) . \quad (3.22)$$

Then a straightforward calculation shows:

$$\begin{aligned} [F_p, F_q] &= \sum_{n \in Z} \left(S_n^a S_{p+q-n}^a - \tilde{S}_{-n-p}^a \tilde{S}_{n-q}^a \right) , \\ &= \delta_{p+q,0} \sum_{n \in Z} \left(S_n^a S_{-n}^a - \tilde{S}_{-n-p}^a \tilde{S}_{n+p}^a \right) . \end{aligned} \quad (3.23)$$

When $p + q \neq 0$, the first term in the first step can be evaluated as: $\sum_{n \in Z} S_n^a S_{p+q-n}^a = \frac{1}{2} \sum_{n \in Z} \{S_n^a, S_{p+q-n}^a\} = 0$. Similarly the second term also gives zero. However, when $p + q = 0$, each term gives $\frac{d}{2} \sum_{n \in Z} 1 \rightarrow \infty$ and therefore the result is ambiguous. Since the result should be a c-number the best way to calculate it is to compute the vacuum expectation value by directly using (3.22). Considering the MNO-vacua $|\eta\rangle$ as defined in appendix A it is straightforward to show that,

$$\langle \eta' | [F_p, F_{-p}] | \eta \rangle = -dp \langle \eta' | \eta \rangle . \quad (3.24)$$

Using this one gets the result obtained in the PNO-prescription in (3.21).

4 Conclusion

In conclusion, we have suggested an explanation of how to reconcile the observation (1.1) made by KY in [5] with the expectation that the right vacuum of the theory should be the one associated to MNO. The key point is that the fields appearing in the space-time effective theory are in general related to the corresponding sigma-model couplings through some field redefinition. For our restricted off-shell ansatz this field redefinition can simply be taken to be identity if the interaction term is ordered according to PNO. This is supported by showing that the resulting EM tensor reproduces the correct physical spectrum for the on-shell background. A consequence of such a definition of the EM tensor is that the Virasoro anomaly terms need to be ordered according to PNO before taking the $\epsilon \rightarrow 0$ limit. This fixes certain ambiguity in computing the Virasoro anomaly in the present method of computation and correctly reproduces the effective equation of motion. We also pointed out that this prescription is not in contradiction with the fact that the right vacuum is the one associated to MNO. It will be interesting to understand the relevance of the normal ordering prescriptions for the kind of analysis done here and in [5] for more complicated backgrounds.

¹¹I thank R. Shankar for a useful discussion on this point.

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A Semi-light-cone quantization in R-R plane-wave

Here we will review the basic steps of the relevant semi-light-cone quantization and then define the two normal ordering prescriptions of our interest. We will follow the same notations and conventions for the space-time and world-sheet indices and gamma matrices as in [7]. Furthermore, we take the following SO(8) decomposition of the 16-dimensional gamma matrices: $\gamma^0 = -\bar{\gamma}^0 = \mathbb{I}_{16}$, $\gamma^9 = \bar{\gamma}^9 = \begin{pmatrix} \mathbb{I}_8 & 0 \\ 0 & -\mathbb{I}_8 \end{pmatrix}$ and $\gamma^I = \bar{\gamma}^I = \begin{pmatrix} 0 & \sigma_{aa}^I \\ \bar{\sigma}_{\dot{a}\dot{a}}^I & 0 \end{pmatrix}$, where $I = 1, \dots, d(=8)$ and $\sigma^I = (\bar{\sigma}^I)^T$ are the real SO(8) gamma matrices.

The classical world-sheet lagrangian density \mathcal{L} is given by,

$$\begin{aligned} 2\pi\alpha'\mathcal{L} = & -\frac{1}{2}\sqrt{-g}g^{ab}(2\partial_a X^+ \partial_b X^- - \mu^2 X_I^2 \partial_a X^+ \partial_b X^+ + \partial_a X^I \partial_b X^I) \\ & -i\sqrt{-g}g^{ab}\partial_b X^+ (\bar{\theta}\bar{\gamma}^- \partial_a \theta + \theta\bar{\gamma}^- \partial_a \bar{\theta} + 2i\mu\partial_a X^+ \bar{\theta}\bar{\gamma}^- \Pi\theta) \\ & +i\epsilon^{ab}\partial_a X^+ (\theta\bar{\gamma}^- \partial_b \theta + \bar{\theta}\bar{\gamma}^- \partial_b \bar{\theta}), \end{aligned} \quad (\text{A.1})$$

where the complex Weyl spinor θ is related to real Majorana-Weyl spinors θ^A ($A = 1, 2$) satisfying the kappa gauge condition $\bar{\gamma}^+ \theta^A = 0$ in the same way as in [7].

The theory has constraints and after going through Dirac's procedure [5] the non-trivial equal time commutators (obtained from Dirac brackets) among the full set of basic operators are given by,

$$[X^\mu(\sigma), P_\nu(\sigma')] = i\delta_\nu^\mu \delta(\sigma - \sigma'), \quad \{S_a^A(\sigma), S_b^B(\sigma')\} = 2\pi\delta^{AB} \delta_{ab} \delta(\sigma - \sigma'), \quad (\text{A.2})$$

where $P_\mu(\sigma) = \frac{\delta S}{\delta(\partial_\tau X^\mu(\sigma))}$, S being the world-sheet action and the classical definition of $S^A(\sigma)$ is given by,

$$S^A(\sigma) = \sqrt{\frac{2\pi^A(\sigma)}{\sqrt{\alpha'}}} \bar{\gamma}^- \theta^A(\sigma), \quad (\text{A.3})$$

where $\pi^1 = \Pi^+$, $\pi^2 = \tilde{\Pi}^+$ and,¹²

$$\Pi^\mu = \sqrt{\frac{\pi}{T}} \eta^{\mu\nu} P_\nu - \sqrt{\pi T} \partial_\sigma X^\mu, \quad \tilde{\Pi}^\mu = \sqrt{\frac{\pi}{T}} \eta^{\mu\nu} P_\nu + \sqrt{\pi T} \partial_\sigma X^\mu, \quad (\text{A.4})$$

where $T = \frac{1}{2\pi\alpha'}$ is the string tension.

Next we define MNO and PNO. To do that we first mode expand various fields in the following way (we have renamed: $S^1 \rightarrow S$, $S^2 \rightarrow \tilde{S}$):

$$\begin{aligned} \Pi^\mu(\sigma) &= \sum_n \Pi_n^\mu e^{in\sigma}, & \tilde{\Pi}^\mu(\sigma) &= \sum_n \tilde{\Pi}_n^\mu e^{-in\sigma}, \\ S^a(\sigma) &= \sum_n S_n^a e^{in\sigma}, & \tilde{S}^a(\sigma) &= \sum_n \tilde{S}_n^a e^{-in\sigma}, \end{aligned} \quad (\text{A.5})$$

¹²Our definitions of Π^μ and $\tilde{\Pi}^\mu$ have been interchanged with respect to that in [12].

such that the equal time commutators read: $[\Pi_m^\mu, \Pi_n^\nu] = [\tilde{\Pi}_m^\mu, \tilde{\Pi}_n^\nu] = m\eta^{\mu\nu}\delta_{m+n,0}$, $\{S_m^a, S_n^b\} = \{\tilde{S}_m^a, \tilde{S}_n^b\} = \delta_{ab}\delta_{m+n}$. Then we define two sets of vacuum states $|\eta\rangle$ and $|\eta\rangle'$ ($\eta = I, \dot{a}$ refer to the vector and conjugate spinor representations of the fermion zero modes respectively) such that they are annihilated by the following sets of operators:

$$\begin{aligned} \{\Pi_n^\mu, \tilde{\Pi}_n^\mu\}|\eta\rangle &= 0, & \forall n \geq 0, & & \{S_n^a, \tilde{S}_n^a\}|\eta\rangle, & \forall n > 0, \\ \{\Pi_n^\mu, \tilde{\Pi}_{-n}^\mu\}|\eta\rangle' &= 0, & \forall n \geq 0, & & \{S_n^a, \tilde{S}_{-n}^a\}|\eta\rangle', & \forall n > 0. \end{aligned} \quad (\text{A.6})$$

MNO and PNO are defined to be the oscillator normal ordering with respect to $|\eta\rangle$ and $|\eta\rangle'$ respectively.

B Regularization procedure and propagators

Here we will discuss the regularization procedure that we adopt for our explicit computations. Any local operator $\mathcal{O}(\Pi^\mu(\sigma), \tilde{\Pi}^\mu(\sigma), S^a(\sigma), \tilde{S}^a(\sigma))$ constructed out of the phase space variables is regularized by replacing the arguments by their regularized versions: $\mathcal{O}_\epsilon(\Pi_\epsilon^\mu(\sigma), \tilde{\Pi}_\epsilon^\mu(\sigma), S_\epsilon^a(\sigma), \tilde{S}_\epsilon^a(\sigma))$ where,

$$\begin{aligned} \Pi_\epsilon^\mu(\sigma) &= \Pi_{(+)}^\mu(\sigma + i\epsilon/2) + \Pi_{(-)}^\mu(\sigma - i\epsilon/2), \\ \tilde{\Pi}_\epsilon^\mu(\sigma) &= \tilde{\Pi}_{(+)}^\mu(\sigma - i\epsilon/2) + \tilde{\Pi}_{(-)}^\mu(\sigma + i\epsilon/2), \\ S_\epsilon^a(\sigma) &= S_0^a + S_{(+)}^a(\sigma + i\epsilon/2) + S_{(-)}^a(\sigma - i\epsilon/2), \\ \tilde{S}_\epsilon^a(\sigma) &= \tilde{S}_0^a + \tilde{S}_{(+)}^a(\sigma - i\epsilon/2) + \tilde{S}_{(-)}^a(\sigma + i\epsilon/2), \end{aligned} \quad (\text{B.1})$$

where the subscripts (\pm) refer to the annihilation and creation parts according to MNO. For example [5], $\{S^a(\sigma), S^b(\sigma')\} \rightarrow \{S_\epsilon^a(\sigma), S_\epsilon^b(\sigma')\} = \delta^{ab} (d(q, \epsilon) + d(q^{-1}, \epsilon) - 1) = 2\pi\delta^{ab}\delta_\epsilon(\Delta)$, where $\Delta = \sigma - \sigma'$, $q = e^{i\Delta}$ and $d(q, \epsilon) = \sum_{n \geq 0} q^n e^{-n\epsilon} = \frac{1}{1 - qe^{-\epsilon}}$. Important identities involving $d(q, \epsilon)$ have been summarized in appendix A of [5].

To compute the equal time propagator for the bosonic fields we first mode expand: $X^\mu(\sigma) = \sum_n X_n^\mu e^{in\sigma}$. Then using $X_n^I = \frac{i}{2n\sqrt{\pi T}} (\Pi_n^I - \tilde{\Pi}_{-n}^I)$, $\forall n \neq 0$ we regularize the bosonic fields as,

$$X_\epsilon^\mu(\sigma) = X_0^\mu - \frac{1}{2\sqrt{\pi T}} \int d\sigma \Pi_\epsilon^\mu(\sigma) + \frac{1}{2\sqrt{\pi T}} \int d\sigma \tilde{\Pi}_\epsilon^\mu(\sigma), \quad (\text{B.2})$$

where we use indefinite integrals. Finally, a straightforward computation gives the following expected results,

$$X_\epsilon^\mu(\sigma) X_\epsilon^\nu(\sigma') = \overset{\times}{\times} X_\epsilon^\mu(\sigma) X_\epsilon^\nu(\sigma') \overset{\times}{\times} =: X_\epsilon^\mu(\sigma) X_\epsilon^\nu(\sigma') : + \eta^{\mu\nu} D_\epsilon(\Delta), \quad (\text{B.3})$$

where $: \ :$ and $\overset{\times}{\times}$ denote MNO and PNO respectively and the propagator $\eta^{\mu\nu} D_\epsilon(\Delta)$ is given by,

$$\begin{aligned} D_\epsilon(\Delta) &= \frac{i}{4\pi T} \int d\Delta (d(q, \epsilon) - d(q^{-1}, \epsilon)), \\ &= -\frac{1}{4\pi T} \ln(\Delta^2 + \epsilon^2). \end{aligned} \quad (\text{B.4})$$

To derive the second line we expand $d(q, \epsilon)$ in small $\epsilon - i\Delta$ to first show $\partial_\Delta D_\epsilon(\Delta) = -\frac{1}{2\pi T} \frac{\Delta}{\Delta^2 + \epsilon^2}$, then integrate this result. Notice that the first equality in (B.3) indicates that given any classical function $f(x)$, the corresponding unordered operator $f(X(\sigma))$ is same as the one ordered according to PNO.

For the fermionic sector one finds,

$$\begin{aligned} S^a(\sigma)S^b(\sigma') &= :S^a(\sigma)S^b(\sigma') : + \delta^{ab} (d(q, \epsilon) - 1) , \\ \tilde{S}^a(\sigma)\tilde{S}^b(\sigma') &= : \tilde{S}^a(\sigma)\tilde{S}^b(\sigma') : + \delta^{ab} (d(q^{-1}, \epsilon) - 1) , \\ &= \times \tilde{S}^a(\sigma)\tilde{S}^b(\sigma') \times + \delta^{ab} (d(q, \epsilon) - 1) , \end{aligned} \tag{B.5}$$

The analogue of (B.3) in this case is given by,

$$\times \tilde{S}^a(\sigma)\tilde{S}^b(\sigma') \times =: \tilde{S}^a(\sigma)\tilde{S}^b(\sigma') : + 4\pi iT \delta^{ab} \partial_\Delta D_\epsilon(\Delta) . \tag{B.6}$$

C Normal ordering constant

Here we will compute the normal ordering constant as shown in eqs. (2.2) which is required to compute the physical spectrum in subsection 2.2. The Virasoro modes are defined by the following mode expansions: $\mathcal{T}(\sigma) = \sum_n L_n e^{in\sigma}$, $\tilde{\mathcal{T}}(\sigma) = \sum_n \tilde{L}_n e^{-in\sigma}$. As usual, only the Virasoro zero modes have the normal ordering ambiguity. It is clear from the definition of EM tensor given in section 2 that the normal ordering constants do not receive any contribution from the interaction part $\delta\mathcal{T}$. Therefore below we will simply restrict ourselves to the flat background.

Let us consider the expression of $\mathcal{T}^{(0)}(\sigma)$ as given in (2.2) with an arbitrary normal ordering constant a instead of 1. One can then directly compute the equal time commutator to get the following result:

$$\begin{aligned} [\mathcal{T}^{(0)}(\sigma), \mathcal{T}^{(0)}(\sigma')] &= \pi i \left[\frac{3d/2 + 2 - 24\xi}{6} \delta'''(\Delta) - \left(4\mathcal{T}^{(0)}(\sigma) - 4a - \frac{1}{3} \right) \delta'(\Delta) \right. \\ &\quad \left. - 2\partial_\sigma \mathcal{T}^{(0)}(\sigma) \delta(\Delta) \right] . \end{aligned} \tag{C.1}$$

In terms of the modes $L_n^{(0)} = \oint \mathcal{T}^{(0)}(\sigma) e^{-in\sigma}$ it corresponds to:

$$[L_m^{(0)}, L_n^{(0)}] = (m - n)L_{m+n}^{(0)} + \left[\frac{3d/2 + 2 - 24\xi}{12} m^3 - \left(\frac{1}{6} + 2a \right) m \right] \delta_{m+n} , \tag{C.2}$$

Therefore choosing,

$$a = 1, \quad \xi = -\frac{1}{2}, \tag{C.3}$$

one recovers the Virasoro algebra in the standard form: $[L_m^{(0)}, L_n^{(0)}] = (m - n)L_{m+n}^{(0)} + \frac{c}{12}(m^3 - m)\delta_{m+n}$, with $c = 26$. Similar results hold for the left moving sector as well.

D Proof of eqs. (3.9)

Here we will prove the results in (3.9). Let us first define the following operators,

$$\begin{aligned}
 l &= : \Pi^+ \Pi^- : + \xi \partial_\sigma^2 \ln \Pi^+, & t &= \frac{1}{2} : \Pi^I \Pi^I :, & s &= -\frac{i}{2} (S \partial_\sigma S), \\
 \tilde{l} &= : \tilde{\Pi}^+ \tilde{\Pi}^- : + \xi \partial_\sigma^2 \ln \tilde{\Pi}^+, & \tilde{t} &= \frac{1}{2} : \tilde{\Pi}^I \tilde{\Pi}^I :, & \tilde{s} &= \frac{i}{2} (\tilde{S} \partial_\sigma \tilde{S}),
 \end{aligned} \tag{D.1}$$

such that $\mathcal{T}^{(0)} = l + t + s + 1$ and $\tilde{\mathcal{T}}^{(0)} = \tilde{l} + \tilde{t} + \tilde{s} + 1$. Using the regularization procedure discussed in appendix B and the commutation relations below eqs. (A.5) one derives the following results:

$$\begin{aligned}
 [l(\sigma), \delta \mathcal{T}_B(\sigma')] &= \pi i \Pi^+(\sigma) \tilde{\Pi}^+(\sigma') K(\vec{X}(\sigma')) \delta'(\Delta), \\
 [l(\sigma), \delta \mathcal{T}_F(\sigma')] &= \frac{\mu}{2} \sqrt{\frac{\pi}{T}} \Pi^+(\sigma) \sqrt{\frac{\tilde{\Pi}^+(\sigma')}{\tilde{\Pi}^+(\sigma')}} (S \Sigma \tilde{S}(\sigma')) \delta'(\Delta), \\
 [s(\sigma), \delta \mathcal{T}_F(\sigma')] &= \frac{\mu}{2} \sqrt{\frac{\pi}{T}} \chi(\sigma') \left[(S(\sigma) \Sigma \tilde{S}(\sigma')) \delta'(\Delta) - (\partial_\sigma S \Sigma \tilde{S}(\sigma)) \delta(\Delta) \right]. \\
 [\tilde{l}(\sigma), \delta \mathcal{T}_B(\sigma')] &= -\pi i \Pi^+(\sigma') \tilde{\Pi}^+(\sigma) K(\vec{X}(\sigma')) \delta'(\Delta), \\
 [\tilde{l}(\sigma), \delta \mathcal{T}_F(\sigma')] &= -\frac{\mu}{2} \sqrt{\frac{\pi}{T}} \tilde{\Pi}^+(\sigma) \sqrt{\frac{\Pi^+(\sigma')}{\tilde{\Pi}^+(\sigma')}} (S \Sigma \tilde{S}(\sigma')) \delta'(\Delta), \\
 [\tilde{s}(\sigma), \delta \mathcal{T}_F(\sigma')] &= -\frac{\mu}{2} \sqrt{\frac{\pi}{T}} \chi(\sigma') \left[(S(\sigma') \Sigma \tilde{S}(\sigma)) \delta'(\Delta) - (S \Sigma \partial_\sigma \tilde{S}(\sigma)) \delta(\Delta) \right],
 \end{aligned} \tag{D.2}$$

where we have suppressed the cutoff ϵ . Using the above results and the identity:

$$\mathcal{O}(\sigma') \delta'(\Delta) = \mathcal{O}(\sigma) \delta'(\Delta) + \partial_\sigma \mathcal{O}(\sigma) \delta(\Delta), \tag{D.4}$$

one gets,

$$\begin{aligned}
 [l(\sigma) + s(\sigma), \delta \mathcal{T}(\sigma')] - (\sigma \leftrightarrow \sigma') &= -4\pi i \delta \mathcal{T}(\sigma) \delta'(\Delta) - 2\pi i \partial_\sigma \delta \mathcal{T}(\sigma) \delta(\Delta), \\
 [\tilde{l}(\sigma) + \tilde{s}(\sigma), \delta \mathcal{T}(\sigma')] - (\sigma \leftrightarrow \sigma') &= 4\pi i \delta \mathcal{T}(\sigma) \delta'(\Delta) + 2\pi i \partial_\sigma \delta \mathcal{T}(\sigma) \delta(\Delta),
 \end{aligned} \tag{D.5}$$

and,

$$[l(\sigma) + s(\sigma), \delta \mathcal{T}(\sigma')] + [\delta \mathcal{T}(\sigma), \tilde{l}(\sigma') + \tilde{s}(\sigma')] = \pi i \chi^2(\sigma) \partial_\sigma K(\vec{X}(\sigma)) \delta(\Delta). \tag{D.6}$$

Using (D.5) in the first two equations in (3.3) one gets the first two equations in (3.9). Using (D.6) in the last equation of (3.3) one gets the last equation in (3.9).

E Problem with PNO-vacuum

In subsection 2.2 we computed the correct physical spectrum by expanding the string field in the transverse Hilbert space \mathcal{H}_T which is a Fock space built over the MNO-vacua $|p, \eta\rangle$.

In the context of bosonic string theory it was shown in [12] that the spectrum contains negative dimensions, as expected, if we consider the PNO-vacua $|p, \eta\rangle'$ instead. Below we will briefly indicate that this is still true in the case of superstrings.

Let us try to see what changes we need to make in the analysis in subsection 2.2. Recall that $\delta\mathcal{T}$ has already been ordered according to PNO. The free parts in (2.2) can be reordered according to PNO. But this does not change the final expression as the difference between MNO and PNO ordered expressions is a c-number which cancels between the bosons and fermions. Therefore the operator expressions in (2.12) remain the same and so does the problem of diagonalizing $\mathcal{S}_2^{\neq 0}$. Hence we arrive at the same expression as in (2.14). The only difference is that now we expand the string field $|\Psi_\perp\rangle$ in terms of the basis states $|\{N\}, p, \eta\rangle'$ which can be obtained by replacing $\tilde{\Pi}_{-n}^I \rightarrow \tilde{\Pi}_n^I$ and $\tilde{S}_{-n}^a \rightarrow \tilde{S}_n^a$ in (2.9). Moreover the transformation (2.13) is not a Bogoliubov transformation any more as it does not mix up the creation and annihilation operators (as defined in PNO). The same is true for the bosonic counterpart as well [12]. Therefore $|p, \eta\rangle'$ remains a vacuum with respect to the new oscillators, i.e.:

$$\left\{ \alpha_n^I, \tilde{\alpha}_{-n}^I, U_n^a, \tilde{U}_{-n}^a \right\} |p, \eta\rangle' = 0. \tag{E.1}$$

Writing the left moving part of $\mathcal{S}_2^{\neq 0}$ as $\sum_{n>0} \left[\tilde{\alpha}_n^I \tilde{\alpha}_{-n}^I - w_n \tilde{U}_n^a \tilde{U}_{-n}^a \right]$ it is now easy to see that both the bosonic and fermionic parts have negative eigenvalues in the Hilbert space spanned by the states $|\{N\}, p, \eta\rangle'$.

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